SFGate.com Return to regular view
Print This Article

Vanishing act has math pros re-solving riddle

- Dennis Overbye, New York Times
Sunday, August 20, 2006

Grisha Perelman, where are you?

Three years ago, a Russian mathematician by the name of Grigory Perelman, a.k.a. Grisha, in St. Petersburg, announced that he had solved a famous and intractable mathematical problem, known as the Poincare conjecture, about the nature of space.

After posting a few short papers on the Internet and making a whirlwind lecture tour of the United States, Perelman disappeared back into the Russian woods in the spring of 2003, leaving the world's mathematicians to pick up the pieces and decide if he was right.

Now they say they have finished his work, and the evidence is circulating among scholars in the form of three book-length papers with about 1,000 pages of dense mathematics and prose among them.

As a result there is a growing feeling, a cautious optimism that they have finally achieved a landmark not just of mathematics, but of human thought.

"It's really a great moment in mathematics," said Bruce Kleiner of Yale, who has spent the past three years helping to explicate Perelman's work. "It could have happened 100 years from now, or never."

In a speech at a conference in Beijing this summer, Shing-Tung Yau of Harvard said the understanding of three-dimensional space brought about by Poincare's conjecture could be one of the major pillars of math in the 21st century.

Quoting Poincare himself, Yau said, "Thought is only a flash in the middle of a long night, but the flash that means everything."

But at the moment of his putative triumph, Perelman is nowhere in sight. He is an odds-on favorite to win a Fields Medal, math's version of the Nobel Prize, when the International Mathematics Union convenes in Madrid on Tuesday. But there is no indication he will show up.

Also left hanging, for now, is $1 million offered by the Clay Mathematics Institute in Cambridge, Mass., for the first published proof of the conjecture, one of seven outstanding questions for which they offered a ransom back at the beginning of the millennium.

"It's very unusual in math that somebody announces a result this big and leaves it hanging," said John Morgan of Columbia, one of the scholars who has also been filling in the details of Perelman's work.

Mathematicians have been waiting for this result for more than 100 years, ever since the French polymath Henri Poincare posed the problem in 1904. And they acknowledge that it may be another 100 years before its full implications for math and physics are understood. For now, they say, it is just beautiful, like art or a challenging new opera.

Morgan said the excitement came not from the final proof of the conjecture, which everybody felt was true, but the method, "finding deep connections between what were unrelated fields of mathematics."

William Thurston of Cornell, the author of a deeper conjecture that includes Poincare's and that is now apparently proved, said, "Math is really about the human mind, about how people can think effectively, and why curiosity is quite a good guide," explaining that curiosity is tied in some way with intuition.

"You don't see what you're seeing until you see it," Thurston said, "but when you do see it, it lets you see many other things."

Depending on who is talking, Poincare's conjecture can sound either daunting or deceptively simple. It asserts that if any loop in a certain kind of three-dimensional space can be shrunk to a point without ripping or tearing either the loop or the space, the space is equivalent to a sphere.

The conjecture is fundamental to topology, the branch of math that deals with shapes, sometimes described as geometry without the details. To a topologist, a sphere, a cigar and a rabbit's head are all the same because they can be deformed into one another. Likewise, a coffee mug and a doughnut are also the same because each has one hole, but they are not equivalent to a sphere.

In effect, what Poincare suggested was that anything without holes has to be a sphere. The one qualification was that this "anything" had to be what mathematicians call compact, or closed, meaning that it has a finite extent: no matter how far you strike out in one direction or another, you can get only so far away before you start coming back, the way you can never get more than 12,500 miles from home on Earth.

In the case of two dimensions, like the surface of a sphere or a doughnut, it is easy to see what Poincare was talking about: Imagine a rubber band stretched around an apple or a doughnut; on the apple, the rubber band can be shrunk without limit, but on the doughnut it is stopped by the hole.

With three dimensions, it is harder to discern the overall shape of something; we cannot see where the holes might be. "We can't draw pictures of 3-D spaces," Morgan said, explaining that when we envision the surface of a sphere or an apple, we are really seeing a two-dimensional object embedded in three dimensions. Indeed, astronomers are still arguing about the overall shape of the universe, wondering if its topology resembles a sphere, a bagel or something even more complicated.

Poincare's conjecture was subsequently generalized to any number of dimensions, but in fact the three-dimensional version has turned out to be the most difficult of all cases to prove. In 1960, Stephen Smale, now at the Toyota Technological Institute at Chicago, proved that it is true in five or more dimensions and was awarded a Fields Medal. In 1983, Michael Freedman, now at Microsoft, proved that it is true in four dimensions and also won a Fields.

"You get a Fields Medal for just getting close to this conjecture," Morgan said.

In the late 1970s, Thurston extended Poincare's conjecture, showing that it was only a special case of a more powerful and general conjecture about three-dimensional geometry, namely that any space can be decomposed into a few basic shapes.

Mathematicians had known since the time of Georg Friedrich Bernhard Riemann, in the

19th century, that in two dimensions there are only three possible shapes: flat like a sheet of paper, closed like a sphere, or curved uniformly in two opposite directions like a saddle or the flare of a trumpet. Thurston suggested that eight different shapes could be used to make up any three-dimensional space.

"Thurston's conjecture almost leads to a list," Morgan said. "If it is true," he added, "Poincare's conjecture falls out immediately." Thurston won a Fields in 1986.

Topologists have developed an elaborate set of tools to study and dissect shapes, including imaginary cutting and pasting, which they refer to as "surgery," but they were not getting anywhere for a long time.

In the early 1980s, Richard Hamilton of Columbia suggested a new technique, called the Ricci flow, borrowed from the kind of mathematics that underlies Einstein's general theory of relativity and string theory, to investigate the shapes of spaces.

Hamilton's technique makes use of the fact that for any kind of geometric space there is a formula called the metric, which determines the distance between any pair of nearby points. Applied mathematically to this metric, the Ricci flow acts like heat, flowing through the space in question, smoothing and straightening all its bumps and curves to reveal its essential shape, the way a hair dryer shrink-wraps plastic.

Hamilton succeeded in showing that certain generally round objects, like a head, would evolve into spheres under this process, but the fates of more complicated objects were problematic. As the Ricci flow progressed, kinks and neck pinches, places of infinite density known as singularities, could appear, pinch off and even shrink away. Topologists could cut them away, but there was no guarantee that new ones would not keep popping up forever.

"All sorts of things can potentially happen in the Ricci flow," said Robert Greene, a mathematician at UCLA. Nobody knew what to do with these things, so the result was a logjam.

It was Perelman who broke the logjam. He was able to show that the singularities were all friendly. They turned into spheres or tubes. Moreover, they did it in a finite time once the Ricci flow started. That meant topologists could, in their fashion, cut them off and allow the Ricci process to continue to its end, revealing the topologically spherical essence of the space in question, and thus proving the conjectures of both Poincare and Thurston.

Perelman's first paper, promising "a sketch of an eclectic proof," came as a bolt from the blue when it was posted on the Internet in November 2002. "Nobody knew he was working on the Poincare conjecture," said Michael Anderson of the State University of New York in Stony Brook.

Perelman had already established himself as a master of differential geometry, the study of curves and surfaces, which is essential to, among other things, relativity and string theory. Born in St. Petersburg in 1966, he distinguished himself as a high school student by winning a gold medal with a perfect score in the International Mathematical Olympiad in 1982. After getting a doctorate from St. Petersburg State, he joined the Steklov Institute of Mathematics at St. Petersburg.

In a series of postdoctoral fellowships in the United States in the early 1990s, Perelman impressed his colleagues as "a kind of unworldly person," in the words of Greene of UCLA -- friendly but shy and not interested in material wealth.

"He looked like Rasputin, with long hair and fingernails," Greene said.

Asked about Perelman's pleasures, Anderson said that he talked a lot about hiking in the woods near St. Petersburg looking for mushrooms.

Perelman returned to those woods and the Steklov Institute in 1995, spurning offers from Stanford and Princeton, among others. In 1996 he added to his legend by turning down a prize for young mathematicians from the European Mathematics Society.

Until his papers on Poincare started appearing, some friends thought Perelman had left mathematics. Although they were so technical and abbreviated that few mathematicians could read them, they quickly attracted interest among experts. In the spring of 2003, Perelman came back to the United States to give a series of lectures at Stony Brook and the Massachusetts Institute of Technology, and also spoke at Columbia, New York University and Princeton.

But once he was back in St. Petersburg, he did not respond to further invitations. The e-mail gradually ceased.

"He came once, he explained things, and that was it," Anderson said. "Anything else was superfluous."

Recently, Perelman is said to have resigned from Steklov. E-mail messages addressed to him and to the Steklov Institute went unanswered.

In his absence, others have taken the lead in trying to verify and disseminate his work.

Kleiner of Yale and John Lott of the University of Michigan have assembled a monograph annotating and explicating Perelman's proof of the two conjectures.

Morgan of Columbia and Gang Tian of Princeton have followed Perelman's prescription to produce a more detailed 473-page step-by-step proof only of Poincare's Conjecture. "Perelman did all the work," Morgan said. "This is just explaining it."

Both works were supported by the Clay institute, which has posted them on its Web site, www.claymath.org. Meanwhile, Huai-Dong Cao of Lehigh University and Xi-Ping Zhu of Zhongshan University in Guangzhou, China, have published their own 318-page proof of both conjectures in the Asian Journal of Mathematics (www.ims.cuhk.edu.hk/).

Although these works were all hammered out in the midst of discussion and argument by experts, in workshops and lectures, they are about to receive even stricter scrutiny and perhaps crossfire. "Caution is appropriate," said Kleiner, because the Poincare conjecture is not just famous, but important.

James Carlson, president of the Clay Institute, said the appearance of these papers had started the clock ticking on a two-year waiting period mandated by the rules of the Clay Millennium Prize. After two years, he said, a committee will be appointed to recommend a winner or winners if it decides the proof has stood the test of time.

"There is nothing in the rules to prevent Perelman from receiving all or part of the prize," Carlson said, saying that Perelman and Hamilton had obviously made the main contributions to the proof.

In a lecture at MIT in 2003, Perelman described himself "in a way" as Hamilton's disciple, although they had never worked together. Hamilton, who got his doctorate from Princeton in 1966, is too old to win the Fields medal, which is given only up to the age of 40, but he is slated to give the major address about the Poincare conjecture in Madrid this week. He did not respond to requests for an interview.

Allowing that Perelman, should he win the Clay Prize, might refuse the honor, Carlson said the institute could decide instead to use award money to support Russian mathematicians, the Steklov Institute or even the Math Olympiad.

Anderson said that to some extent the new round of papers already represented a kind of peer review of Perelman's work. "All these together make the case pretty clear," he said. "The community accepts the validity of his work. It's commendable that the community has gotten together."

Page A - 2
URL: http://sfgate.com/cgi-bin/article.cgi?file=/c/a/2006/08/20/MNG71KJJPK1.DTL

©2006 San Francisco Chronicle