Tutorato 10 - ICA
Soluzioni

a) Calcolare i seguenti limiti utilizzando lo sviluppo di Taylor:

1. $\lim_{x\to 0} \frac{(\sin^2 x - \log\cos x)\log(1+\sin x)}{x\sin x\sin 2x}$

$$= \lim_{x\to 0} \frac{[(x+o(x^2))^2 - \log(1-\frac{x^2}{2}+o(x^3))][\log(1+x+o(x^2))]}{x(x+o(x^2))(2x+o(x^2))} =$$

$$= \lim_{x\to 0} \frac{[x^2+o(x^3)+\frac{x^2}{2}+o(x^2)][x+o(x)]}{2x^3+o(x^4)} =$$

$$= \lim_{x\to 0} \frac{[\frac{3}{2}x^2+o(x^2)][x+o(x)]}{2x^3+o(x^4)} = \lim_{x\to 0} \frac{\frac{3}{2}x^3+o(x^3)}{2x^3+o(x^4)} = \frac{3}{4}$$

2. $\lim_{x\to 0} \frac{\log(1+x\arctan x) - e^{x^2}+1}{\sqrt{1+2x^4}-1}$

$$= \lim_{x\to 0} \frac{\log(1+x(x-\frac{x^3}{3}+o(x^4))) - (1+x^2+\frac{x^4}{2}+o(x^4))+1}{1+\frac{1}{2}2x^4+o(x^4)-1} =$$

$$= \lim_{x\to 0} \frac{\log(1+x^2-\frac{x^4}{3}+o(x^5)) - x^2 - \frac{x^4}{2}+o(x^4)}{x^4+o(x^4)} =$$

$$= \lim_{x\to 0} \frac{x^2-\frac{x^4}{3}+o(x^5)-\frac{1}{2}(x^2-\frac{x^4}{3}+o(x^5))^2+o(x^2-\frac{x^4}{3}+o(x^5))^2-x^2-\frac{x^4}{2}+o(x^4)}{x^4+o(x^4)} =$$

$$= \lim_{x\to 0} \frac{-\frac{x^4}{3}-\frac{1}{2}(x^4+o(x^6))+o(x^4)-\frac{x^4}{2}+o(x^4)}{x^4+o(x^4)} = \lim_{x\to 0} \frac{-\frac{x^4}{3}-\frac{x^4}{2}-\frac{x^4}{2}+o(x^4)}{x^4+o(x^4)} =$$

$$= \lim_{x\to 0} \frac{-\frac{5}{3}x^4+o(x^4)}{x^4+o(x^4)} = -\frac{4}{3}$$