Tutorato di AM1a

Fabrizio Fanelli

1. $\lim_{n\to\infty} n^{-1/2}\log^{\pi}(n)$

   Limite notevole del tipo : $\dfrac{(\log a_n)^{\alpha}}{a_n^{\beta}} \to 0$ se $a_n \to +\infty$ e $\alpha \in \mathbb{R}$, $\beta \in \mathbb{R}^+$

2. $\lim_{n\to\infty} \dfrac{\log^{-2} n}{n^{1/1000}}$ (0).

3. $\lim_{n\to\infty} \dfrac{n^2(\log n)^2}{\sqrt{n^5+1}}$ (0).

4. $\lim_{n\to\infty} \dfrac{n2^n}{3^n}$ (0).

5. $\lim_{n\to\infty} n\log(1/n)$ $-\infty$

6. $\lim_{n\to\infty} \dfrac{n^2 - n! + 2^n}{(2n)! - \log^{-1}(1/n)}$ (0).

7. $\lim_{n\to\infty} \dfrac{n\sin(\log^{2^n}(\sqrt{n^n n!}))}{n^{1+\alpha}}$, dove $\alpha \in \mathbb{R},\ \alpha > 1$ (0).

8. $\lim_{n\to\infty} \dfrac{(n+2)! - n!}{(2n^2+1)n!}$ $(1/2)$.

9. $\lim_{n\to\infty} \dfrac{3n^2 + n\sin(n) + n}{n^2 + n + \cos(n)}$ (3).

10. $\lim_{n\to\infty} \dfrac{(2n)!}{n^n}$ $(+\infty)$.

11. $\lim_{n\to\infty} n - \sqrt{n^2 - n\log(n) + 7n - 1}$ $(+\infty)$.

12. $\lim_{n\to\infty} \sqrt[n]{2^n + 3^n}$ (3).

13. $\lim_{n\to\infty} \sqrt[n]{n!}$ $(+\infty)$.

14. $\lim_{n\to\infty} \left(\sqrt{n+1} - \sqrt{n}\right)\sqrt{n+3}$ (1).

15. $\lim_{n\to\infty} \left[\dfrac{\tan(1/n)}{\tan\left(\frac{2}{n^2}\right)}\right] \quad (+\infty).$

16. $\lim_{n\to\infty} \dfrac{n^2}{n+1} \sin \dfrac{n+1}{n^2} \quad (1).$

17. Dimostrare che : $\lim_{n\to\infty} n^2 \left(1 - \cos \dfrac{1}{n}\right) = 1/2$

    $n^2 \left(1 - \cos \frac{1}{n}\right) = n^2 (2 \sin^2 \frac{1}{2n}) = \frac{1}{2} \left(2n \sin \frac{1}{2n}\right)^2 \to \dfrac{1}{2} 1^2 = \dfrac{1}{2}$

18. $\lim_{n\to\infty} \dfrac{\log(1+n+n^3) - 3\log n}{n(1-\cos\frac{1}{n^2})}$

    Primo passo procedendo come sopra avró che : $n^4 \left(1 - \cos \frac{1}{n^2}\right) \to \frac{1}{2}$

    Secondo passo

    $n^3 \log \left(\dfrac{n^3+n+1}{n^3}\right) = (n+1) \log \left[\left(1 + \dfrac{n+1}{n^3}\right)^{\dfrac{n^3}{n+1}}\right] \to \infty$ poiché

    l'argomento del logaritmo tende ad e

    (limite notevole del tipo $\left(1 + \dfrac{x}{a_n}\right)^{a_n} \to e^x \ \forall x \in \mathbb{R} \ e \ \forall a_n \to +\infty$) .

    Mettendo insieme i due passi noto che il nostro limite è $= +\infty$.

19. $\lim_{n\to\infty} \left(1 + \sin \dfrac{1}{n}\right)^n$

    $= \left[\left(1 + \left|\sin \dfrac{1}{n}\right|\right)^{\dfrac{1}{\sin \frac{1}{n}}}\right]^{n \sin \frac{1}{n}} \to e^1 = e$

20. $\lim_{n\to\infty} n \log_{10}(1 + 2/n)$

    $n \log_{10}(1 + 2/n) = \log_{10}\left[(1 + 2/n)^n\right] \to 2 \log_{10} e$

    (limite notevole $(1 + a/n)^n \to e^a$)

21. $\lim_{n\to\infty} n \ln(1 + 1/n) \quad (1)$

22. $\lim_{n\to\infty} n(e^{(1/n)} - 1)$

    posto $a_n = e^{1/n} - 1$, si ha $e^{1/n} = a_n + 1$ e , passando ai logaritmi ,

$1/n = \log a_n + 1$, allora il nostro limite diventa $\lim_{n\to\infty} \frac{a_n}{\log a_n + 1} =$
$= \lim_{n\to\infty} \frac{1}{1/a_n \log(1+a_n)} = \lim_{n\to\infty} \frac{1}{\log\left[(1+a_n)^{\frac{1}{a_n}}\right]} = \frac{1}{\log e} = 1.$

23. $\lim_{n\to\infty} n \log_a(1+1/n) \qquad \forall a \in \mathbb{R}\ ,\ a>0,\ a \neq 1 \quad (\log_a e)$

24. $\lim_{n\to\infty} (a^{1/n}-1)n \qquad \forall a \in \mathbb{R}\ ,\ a>1\ ,\ a \neq 1 \quad (\frac{1}{\log a})$

25. $\lim_{n\to\infty} n^2\left[\log(1+1/n) + \log(1-1/n)\right] \quad (e^{-1})$

26. $\lim_{n\to\infty} \frac{\sin(3/n)}{\sin(2/n)} \quad (2/3)$